ختم المركز

سلطنة عمان
وزارة التربية والتعليم

حاضر ▢
غائب ▢

امتحان دبلوم التعليم العام للمدارس الخاصة (ثنائية اللغة)
للعام الدراسي ١٤٣٦/١٤٣٥ هـ - ٢٠١٤ / ٢٠١٥ م
الدور الأول - الفصل الدراسي الأول

تنبيه
- المادة: الرياضيات البحتة .
- الأسئلة في (١٤) صفحة.
- زمن الإجابة: ثلاث ساعات.
- الإجابة في الورقة نفسها.

تعليمات وضوابط التقدم للامتحـان:

- الحضور إلى اللجنة قبل عشر دقائق من بدء الامتحان للأهمية.
- إبراز البطاقة الشخصية لمراقب اللجنـة.
- يمنع كتابة رقم الجلوس أو الاسم أو أي بيانات أخرى تدل على شخصية الممتحن في دفتر الامتحان، وإلا ألغي امتحانه.
- يحظر على الممتحنين أن يصطحبوا معهم بمركز الامتحان كتبا دراسية أو كراسات أو مذكرات أو هواتف محمولة أو أجهزة النداء الآلي أو أي شيء له علاقة بالامتحان كما لا يجوز إدخال آلات حادة أو أسلحة من أي نوع كانت أو حقائب يدوية أو آلات حاسبة ذات صفة تخزينية.
- يجب أن يتقيد المتقدمون بالزي الرسمي (الدشداشة البيضاء والمصر أو الكمة للطلاب والدارسين والزي المدرسي للطالبات واللباس العماني للدارسات) ويمنع النقاب داخل المركز ولجان الامتحان.
- لا يسمح للمتقدم المتأخر عن موعد بداية الامتحان بالدخول إلا إذا كان التأخير بعذر قاهر يقبله رئيس المركز وفي حدود عشر دقائق فقط.
- يتم الالتزام بالإجراءات الواردة في دليل الطالب لأداء امتحان شهادة دبلوم التعليم العام.
- يقوم المتقدم بالإجابة عن أسئلة الامتحان المقالية بقلم الحبر (الأزرق أو الأسود).
- يقوم المتقدم بالإجابة عن أسئلة الاختيار من متعدد بتظليل الشكل (▢) وفق النموذج الآتي:

س - عاصمـة سلطنة عمـــان هي:
▢ القاهرة ▢ الدوحة
■ مسقط ▢ أبوظبي

ملاحظة: يتم تظليل الشكل (■) باستخدام القلم الرصاص وعند الخطأ، امسح بعناية لإجراء التغيير.

صحيح ■ **غير صحيح** ▭ ◉ ◒ ☒ ☑

مُسَوَّدَة، لا يتم تصحيحها

Instructions:

1. Programmable calculators are **not** allowed.
2. A list of formulae is provided on the final pages.

Question 1 (28 marks)

There are 14 multiple-choice items worth two marks each.
Shade in the **correct** answer for each of the following items .

1) If $\frac{3x+1}{(x+2)(x+1)} \equiv \frac{A}{(x+2)} + \frac{B}{(x+1)}$, then B equals:

- ☐ -2
- ☐ 4
- ☐ 5
- ☐ 7

2) $\lim_{h \to 0} \frac{f(x+h)-f(x)}{3h} =$

- ☐ $3f'(x)$
- ☐ $\frac{1}{3}f'(x)$
- ☐ $f'(3)$
- ☐ $f'\left(\frac{1}{3}\right)$

3) If $y = ax^2$ has a gradient of 12 at point (2, 12), then a equals:

- ☐ $\frac{1}{6}$
- ☐ $\frac{1}{3}$
- ☐ 3
- ☐ 6

4) If $f'(x) = 6x^2 - 4x^3$, then the number of inflexion point(s) of $f(x)$ equals:

- ☐ 0
- ☐ 1
- ☐ 2
- ☐ 3

Question 1 continued

5) The period of $f(x) = 6\,\text{cosec}\,2(x - \pi)$ is:

- ☐ π
- ☐ 2π
- ☐ 4π
- ☐ 6π

6) If $\sin\theta = \frac{3}{5}$ where $0 < \theta < \frac{\pi}{2}$, then $\sec\theta =$

- ☐ $\frac{3}{4}$
- ☐ $\frac{4}{5}$
- ☐ $\frac{5}{4}$
- ☐ $\frac{4}{3}$

7) If $\sqrt{3}\tan(x) - 1 = 0$ where $0 < x < \frac{\pi}{2}$, then the value of x is:

- ☐ $\frac{\pi}{6}$
- ☐ $\frac{\pi}{4}$
- ☐ $\frac{\pi}{3}$
- ☐ $\frac{\pi}{2}$

8) If $\sec\theta - \tan\theta \neq 0$, then $\frac{1}{\sec\theta - \tan\theta} =$

- ☐ $\sec\theta + \tan\theta$
- ☐ $\sec\theta - \tan\theta$
- ☐ $\cot\theta + \cos\theta$
- ☐ $\cos\theta - \cot\theta$

9) $\int 7x^6\,dx =$

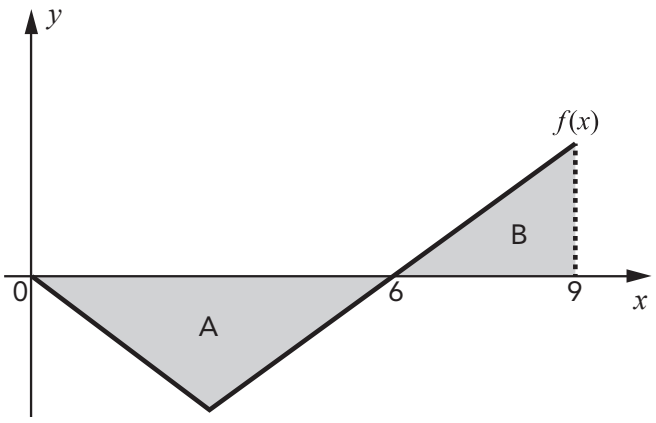
- ☐ $x^6 + c$
- ☐ $x^7 + c$
- ☐ $7x^7 + c$
- ☐ $42x^5 + c$

Do not write in this space

Do not write in this space

Question 1 continued

10) From the figure, if area A = 12 square units and area B = 9 square units, then $\int_0^9 f(x)dx$ equals:



- ☐ −21
- ☐ −3
- ☐ 3
- ☐ 21

11) If $f'(x) = 6x^2$, $f(0) = 1$, then $f(2) =$

- ☐ 25
- ☐ 24
- ☐ 17
- ☐ 16

12) If $\int_0^2 (2x - m)dx = -4$, then $m =$

- ☐ −8
- ☐ −4
- ☐ 4
- ☐ 8

13) The probability that there will be at least one 'tails' in tossing a coin four times is:

- ☐ $\frac{15}{16}$
- ☐ $\frac{7}{8}$
- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{16}$

14) A basket contains 10 green apples and 15 red apples. Ahmed chooses one apple at random and eats it, followed by another one. The probability that Ahmed eats one green and one red apple is:

- ☐ $\frac{6}{25}$
- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{4}$
- ☐ $\frac{23}{25}$

Extended Questions

Write your answer for each of the three questions in the space provided.
Be sure to show all your work and the correct units where applicable.

Question 2 (16 marks)

15) Express $\dfrac{5x^2}{(x^2 + 2x + 1)(x - 2)}$ as a sum of partial fractions. [3 marks]

Question 2 continued

16) Find $\frac{d^2y}{dx^2}$ if $y = x^{\frac{3}{2}} + x + 3$ [4 marks]

Question 2 continued

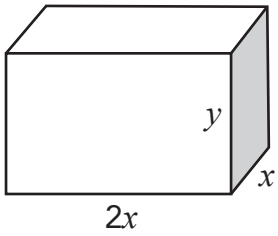
17) a. Find the equation of the tangent to the curve $y = 16 - x^2$ for $x \geq 0$ at the point of intersection of curve y with the x-axis. [3 marks]

b. Given that $y = \frac{1}{3}x^3 - x^2$, find the range of values x for which y is

an increasing function [3 marks]

Question 2 continued

18) A closed rectangular box is to be constructed with a base whose length is twice its width x. If the total surface area is $108\,\text{cm}^2$, calculate the value for x which maximises the box's volume. [3 marks]



Question 3 (14 marks)

19) If $\frac{4x^3 + 10x + 4}{x(2x + 1)} \equiv (2x - 1) + \frac{A}{x} + \frac{B}{(2x + 1)}$, calculate B. [3 marks]

Question 3 continued

20) **Without using a calculator**, calculate the value of:

$\sin 105° \cos 75° + \cos 105° \sin 75°$ [3 marks]

21) **a.** Given that $\cot A = \frac{1}{4}$ and $\cot(A - B) = 8$, calculate the value of $\tan B$.

[3 marks]

Question 3 continued

21) b. Express $\frac{\sqrt{5}}{3}\cos\theta - \frac{\sqrt{5}}{3}\sin\theta$ in the form $R\cos(\theta + \alpha)$ where $R > 0$.

[3 marks]

Question 3 continued

22) Prove that:

$$\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 + \sin x = 1$$ [2 marks]

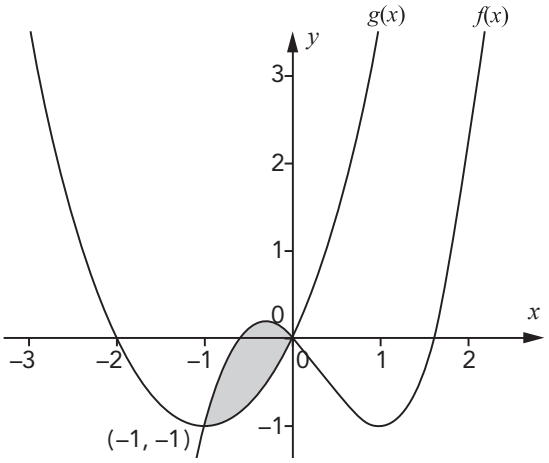
Question 4 (12 marks)

23) Calculate: $\int_0^1 (4x^3 - x + 1)\,dx$ [4 marks]

Question 4 continued

24) Calculate the shaded area of the region bounded by these graphs of

$f(x) = x^3 - x^2 - x$ and $g(x) = x^2 + 2x$ [4 marks]



Question 4 continued

25) A and B are two events such that $P(A) = 0.3$, $P(B) = 0.25$ and $P(A \cap B) = 0.05$.

Draw a Venn diagram to represent the events A and B, and the sample space S.

[2 marks]

26) A and B are two events such that $P(A) = 0.75$, $P(B) = 0.5$ and $P(A \mid B) = 0.4$.

Calculate $(A \cap B')$. [2 marks]

[End of Examination]

List of Formulae (2 sheets)

Differentiation:

1. $y = x^n \qquad \frac{dy}{dx} = nx^{(n-1)} \qquad n \in \mathrm{R}$

2. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

3. $y = kx^n \qquad \frac{dy}{dx} = knx^{(n-1)} \qquad n \in \mathrm{R}$

4. $y = f(x) \pm g(x) \qquad \frac{dy}{dx} = f'(x) \pm g'(x)$

5. $y = kf(x) \qquad \frac{dy}{dx} = kf'(x)$

6. Area and Volume of a cuboid with length, width and height as $l, w,$ and h respectively.

   $Area = 2lw + 2wh + 2lh \qquad Volume = l \times w \times h$

7. Area and Volume of a cylinder with radius $,r,$ and height $,h$.

   $Area = 2\pi rh + 2\pi r^2 \qquad Volume = \pi r^2 h$

8. Area and Volume of a sphere with radius $,r$.

   $Area = 4\pi r^2 \qquad Volume = \frac{4}{3}\pi r^3$

Trigonometry:

Pythagorean Formulas

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A = 1 + \tan^2 A$
3. $\mathrm{cosec}^2 A = 1 + \cot^2 A$

Double Angle Formulas:

1. $\sin 2A = 2\sin A \cos A$
2. $\cos 2A = \cos^2 A - \sin^2 A$

   $\cos 2A = 2\cos^2 A - 1$

   $\cos 2A = 1 - 2\sin^2 A$
3. $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Compound Angle Formulas:

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Half Angle Formulas:

1. $\sin^2 \frac{1}{2}A = \frac{1}{2}(1 - \cos A)$
2. $\cos^2 \frac{1}{2}A = \frac{1}{2}(1 + \cos A)$
3. $\sin^2 A = \frac{1}{2}(1 - \cos 2A)$
4. $\cos^2 A = \frac{1}{2}(1 + \cos 2A)$

The form $a\cos\theta + b\sin\theta$ can be expressed in the form

$R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ where $\mathrm{R} = \sqrt{a^2 + b^2}$, *and* $\tan\alpha = \pm\frac{b}{a}$ *or* $\tan\alpha = \pm\frac{a}{b}$

Integration:

1) $\int x^n dx = \frac{x^{(n+1)}}{n+1} + c, n \neq -1$
2) $\int [f(x) \pm g(x)]\, dx = \int f(x)\, dx \pm \int g(x)\, dx$
3) $\int kf(x)\, dx = k \int f(x)\, dx$
4) Area and volume of solids of revolution

$$Area = \int_a^b f(x)\, dx$$

$$Volume = \pi \int_a^b (f(x))^2\, dx$$

5) Trapezium rule

$$\int_a^b f(x) dx = \frac{h}{2}[y_0 + y_n + 2(y_1 + y_2 + \ldots\ldots\ldots + y_{n-1})$$

Probability:

1) Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2) Conditional Probability:

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \cap B)}{P(B)}$$

3) Multiplication Rule:

$$P(A \cap B) = P(A|B) \times P(B) \text{ or } P(B|A) \times P(A)$$

4) Independent Rule:

A and B are independent if:

$$P(A|B) = P(A) \text{ or } P(B|A) = P(B) \text{ or } P(A \cap B) = P(A) \times P(B)$$

5) Mutually Exclusive Rule:

A and B are Mutually Exclusive if:

$$P(A \cap B) = 0$$

6) $\frac{P(B' \cap A')}{P(A')} = \frac{1 - P(B \cup A)}{1 - P(A)}$
7) $P(A \cap B') = P(A) - P(A \cap B)$

End of Formulae

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MINISTRY OF EDUCATION
GENERAL EDUCATION DIPLOMA
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(Multiple Choice)		
Answer		**Mark**

Answers For Question One:

Question No.	Answer	Page	Taxonomy
1	-2	179	Knowledge
2	$\frac{1}{3}f'(x)$	164	Knowledge
3	3	159	Application
4	1	235	Reasoning
5	π	51	Knowledge
6	$\frac{5}{4}$	63	Application
7	$\frac{\pi}{6}$	52	Application
8	$\sec\theta + \tan\theta$	53	Reasoning
9	$x^7 + c$	166	Knowledge
10	-3	338	Application
11	17	166	Application
12	4	326	Reasoning
13	$\frac{15}{16}$	79	Application
14	$\frac{1}{2}$	95	Application

14 X 2 = 28 marks

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(Extended Questions)		
Answer	**Mark**	**Page/ Taxonomy**
15) $\frac{5x^2}{(x^2+2x+1)(x-2)} \equiv \frac{5x^2}{(x+1)^2(x-2)}$	$\frac{1}{2}$	
$\frac{5x^2}{(x^2+2x+1)(x-2)} \equiv \frac{A}{x-2} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$	$\frac{1}{2}$	
	$\frac{1}{2}$	
$5x^2 \equiv A(x+1)^2 + B(x-2)(x+1) + C(x-2)$ (1)		
putting $x = -1$ in (1)		
$5 = -3C$ $\quad C = -\frac{5}{3}$	$\frac{1}{2}$	184
putting $x = 2$ in (1)	$\frac{1}{2}$	Application
$20 = 9A$ $\quad A = \frac{20}{9}$	$\frac{1}{2}$	
Equating the coefficients of x^2 in (1)		
$5 = A + B \Rightarrow B = 5 - \frac{20}{9}$		
$B = \frac{25}{9}$		
$\frac{5x^2}{(x^2+2x+1)(x-2)} \equiv \frac{20}{9(x-2)} + \frac{25}{9(x+1)} - \frac{5}{3(x+1)^2}$		
16) $y = x^{\frac{3}{2}} + x + 3$	1	159 Knowledge
$y' = \frac{3}{2}x^{\frac{1}{2}} + 1$	1+1	
$y'' = \frac{3}{4}x^{-\frac{1}{2}}$	1	
17)a)To find the intersection points of y with $x-axis$		162 Application
Putting y=0	$\frac{1}{2}$	
$16 - x^2 = 0$		
$(4-x)(4+x) = 0$	$\frac{1}{2}+\frac{1}{2}$	
$x = 4$, *or* $x = -4$ rejected	$\frac{1}{2}$	
So the point of intersection is (4,0)		
$y' = -2x$	$\frac{1}{2}$	
Gradient of tangent at $(x=4) = -2 \times 4 = -8$		
equation $y - y_1 = m(x - x_1)$	$\frac{1}{2}$	
$y - 0 = -8(x-4)$		
$y = -8x + 32$		

(Extended Questions)			
Answer		**Mark**	**Page**
b) y is an increasing function if $\frac{dy}{dx} > 0$ $x^2 - 2x > 0$ $x(x-2) > 0$		$\frac{1}{2}$ 1	231 Application
Value of x | e.g x = -1 | 0 | e.g x = 1 | 2 | e.g x = 4 Sign of y' | ++++++ | 0 | ----- | 0 | ++++++		$\frac{1}{2} + \frac{1}{2}$	
y is an increasing function of x for $x < 0$ and $x > 2$		$\frac{1}{2}$	
18) $V = 2x.x.y$ $V = 2x^2\ y$ (1) $A = 2xy + 4x^2 + 4xy$ $108 = 6xy \quad 4x^2$ $54 = 3xy + 2x^2$ *divide by2* $3xy = 54 - 2x^2$ $y = \frac{54 - 2x^2}{3x}$ *Subistituting y in 1* $V = 2x^2\left(\frac{54 - 2x^2}{3x}\right)$ $V = \frac{2}{3}x(54 - 2x^2)$ $= 36x - \frac{4}{3}x^3$ $V' = 36 - 4x^2$ *put* $V' = 0$, *then* $x^2 = \frac{36}{4} = 9$ $x = 3$, $x = -3$ rejected $V'' = -8x$ $V'' = -8 \times 3 = -24 < 0$ so maximum value when $x = 3$		$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	238 Reasoning

(Extended Questions)		
Answer	**Mark**	**Page**
QUESTION THREE (14 marks		185 Reasoning
19) $\frac{4x^3+10x+4}{x(2x+1)} \equiv (2x-1)+\frac{A}{x}+\frac{B}{2x+1}$		
$\frac{4x^3+10x+4}{x(2x+1)} \equiv (2x-1)+\frac{11x+4}{x(2x+1)}$	1	
$\frac{11x+4}{x(2x+1)} \equiv \frac{A}{x}+\frac{B}{2x+1}$	$\frac{1}{2}$	
$11x+4 = A(2x+1)+Bx$	$\frac{1}{2}$	
$putting\ x = -\frac{1}{2} \quad \frac{-11}{2}+4 = -\frac{1}{2}B \quad \frac{-3}{2} = -\frac{1}{2}B$ $B = 3$	1	
20) $(\sin 105° \cos 75° + \cos 105° \sin 75°)$		70 Knowledge
$= \sin(105° + 75°)$	1	
$=\sin 180°$	1	
$= 0$	1	
21) a) $\tan A = 4, \quad \tan(A-B) = \frac{1}{8}$	$\frac{1}{2}+\frac{1}{2}$	65 Application
$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$		
$\frac{1}{8} = \frac{4-\tan B}{1+4\tan B}$	$\frac{1}{2}+\frac{1}{2}$	
$1+4\tan B = 8(4-\tan B)$	$\frac{1}{2}$	
$12\tan B = 31 \quad , \tan B = \frac{31}{12}$	$\frac{1}{2}$	

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(Extended Questions)

Answer	Mark	Page
b) $\frac{\sqrt{5}}{3}\cos\theta - \frac{\sqrt{5}}{3}\sin\theta \equiv R\cos(\theta+\alpha) = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\frac{\sqrt{5}}{3} = R\cos\alpha \ , \ \frac{\sqrt{5}}{3} = R\sin\alpha$ $R = \sqrt{\left(\frac{\sqrt{5}}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} \ , \ R = \frac{\sqrt{10}}{3}$ $\tan\alpha = 1 \ , \alpha = 45^\circ$ $\frac{\sqrt{5}}{3}\cos\theta - \frac{\sqrt{5}}{3}\sin\theta \equiv \frac{\sqrt{10}}{3}\cos(\theta+45^\circ)$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$	78 Application
22) RHS$=\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2 + \sin x$ $=\cos^2\frac{x}{2} - 2\cos\frac{x}{2}\sin\frac{x}{2} + \sin^2\frac{x}{2} + \sin x$ $= 1 - 2\cos\frac{x}{2}\sin\frac{x}{2} + \sin x = 1 - \sin\left(2\times\frac{x}{2}\right) + \sin x$ $=1=$ LHS	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	69 Reasoning
23) $\int_0^1 (4x^3 - x + 1)\,dx = \left[x^4 - \frac{1}{2}x^2 + x\right]_0^1$ $= \left(1 - \frac{1}{2} + 1\right) - 0 = \frac{3}{2}$	$1+\frac{1}{2}$ $1+1+\frac{1}{2}$	326 Knowledge
24)Area$= \int_{-1}^{0}\big(f(x) - g(x)\big)dx$ $\int_{-1}^{0}[(x^3 - x^2 - x) - (x^2 + 2x)]dx$ $= \int_{-1}^{0}(x^3 - 2x^2 - 3x)\ dx$ $= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{-1}^{0}$ $= 0 - (\frac{1}{4} + \frac{2}{3} - \frac{3}{2})$ $= \frac{7}{12}$ square units.	$\frac{1}{2}$ 1 1 $\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}$	336 Application

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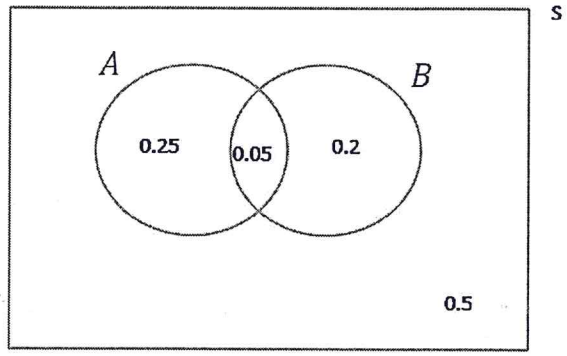
(Extended Questions)		
Answer	**Mark**	**Page**
25) 	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	105 Knowledge
26) $P(A \cap B') = P(A) - P(A \cap B)$ $P(A\|B) = \frac{P(A \cap B)}{P(B)}$ $P(A \cap B) = P(B) \times P(A\|B)$ $= 0.5 \times 0.4 = 0.2$ $P(A \cap B') = P(A) - P(A \cap B)$ $= 0.75 - 0.2 = 0.55$	$\frac{1}{2}+\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$	91 Application

(End of the Marking Guide)