رقم الورقة	
رقم المغلف	

☐ حاضر

☐ غائب

سلطنة عمان
وزارة التربية والتعليم

امتحان شهادة دبلوم التعليم العام للمدارس الخاصة (ثنائية اللغة)

للعام الدراسي ١٤٣٣/١٤٣٤ هـ - ٢٠١٢ / ٢٠١٣ م

الدور الثاني - الفصل الدراسي الأول

تنبيه: • الرياضيات.
• الأسئلة في (١٥) صفحة.
• زمن الإجابة: ثلاث ساعات.
• الإجابة في الورقة نفسها.

تعليمات وضوابط التقدم للامتحـان:

- الحضور إلى اللجنة قبل عشر دقائق من بدء الامتحان للأهمية.
- إبراز البطاقة الشخصية لمراقب اللجنـة.
- يمنع كتابة رقم الجلوس أو الاسم أو أي بيانات أخرى تدل على شخصية الممتحن في دفتر الامتحان، وإلا ألغي امتحانه.
- يحظر على الممتحنين أن يصطحبوا معهم بمركز الامتحان كتبا دراسية أو كراسات أو مذكرات أو هواتف محمولة أو أجهزة النداء الآلي أو أي شيء له علاقة بالامتحان كما لا يجوز إدخال آلات حادة أو أسلحة من أي نوع كانت أو حقائب يدوية أو آلات حاسبة ذات صفة تخزينية.
- يجب أن يتقيد المتقدمون بالزي الرسمي (الدشداشة البيضاء والمصر أو الكمة للطلاب والدارسين والزي المدرسي للطالبات واللباس العماني للدارسات) ويمنع النقاب داخل المركز ولجان الامتحان.
- لا يسمح للمتقدم المتأخر عن موعد بداية الامتحان بالدخول إلا إذا كان التأخير بعذر قاهر يقبله رئيس المركز وفي حدود عشر دقائق فقط.
- يتم الالتزام بالإجراءات الواردة في دليل الطالب لأداء امتحان شهادة دبلوم التعليم العام.
- يقوم المتقدم بالإجابة عن أسئلة الامتحان المقالية بقلم الحبر (الأزرق أو الأسود).
- يقوم المتقدم بالإجابة عن أسئلة الاختيار من متعدد بتظليل الشكل (☐) وفق النموذج الآتي:

س - عاصمـة سلطنة عمـــان هي:
☐ القاهرة ☐ الدوحة
■ مسقط ☐ أبوظبي

ملاحظة: يتم تظليل الشكل (■) باستخدام القلم الرصاص وعند الخطأ، امسح بعناية لإجراء التغيير.

صحيح ■ **غير صحيح** ☐ ◉ ◒ ✗ ✓

Question One (28 marks)

There are 14 multiple–choice items worth two marks each.
Shade the best correct answer for each of the following items.

1. $\lim_{h \to 0} \frac{f(7) - f(7+h)}{2h} =$

- ▢ $-\frac{1}{2}f'(7)$
- ▢ $\frac{1}{2}f'(7)$
- ▢ $f'(7)$
- ▢ $-f'(7)$

2. If $f(x) = ax^2 + 3x$ and $f'(2) = -5$, then the value of a is:

- ▢ -2
- ▢ $-\frac{1}{2}$
- ▢ $\frac{1}{2}$
- ▢ 2

3. If $f(x) = \frac{x^3}{3} + \frac{3x^3}{3} + 9$, then the x-coordinates of the stationary points are:

- ▢ $0, \frac{9}{2}$
- ▢ $0, \frac{-9}{2}$
- ▢ $0, 3$
- ▢ $0, -3$

4. If $\frac{1}{(x+3)(x+2)} = \frac{A}{(x+3)} + \frac{B}{(x+2)}$, then $4A + 6B =$

- ▢ 0
- ▢ 1
- ▢ 2
- ▢ 4

5. Which of trigonometric functions are both have domain $\theta \in R$, $\theta \neq (2n+1)\,90°$:

- ▢ $\cot\theta, \sec\theta$
- ▢ $\sec\theta, \tan\theta$
- ▢ $\operatorname{cosec}\theta, \tan\theta$
- ▢ $\cot\theta, \operatorname{cosec}\theta$

6. If $\operatorname{cosec}\theta = \frac{5}{4}$ and θ is obtuse, then $\tan\theta$ equals:

- ▢ $-\frac{4}{3}$
- ▢ $-\frac{4}{5}$
- ▢ $-\frac{3}{4}$
- ▢ $-\frac{3}{5}$

7. The $\operatorname{cosec}\left(\frac{1}{2}\theta - 5\right) = \sqrt{2}$, $0° < \theta < 180°$, then $\theta =$

- ▢ 20°
- ▢ 25°
- ▢ 80°
- ▢ 100°

8. If $t = \sin\theta$, then $1 - 2t^2 =$

- ▢ $\frac{1}{2}\cos\theta$
- ▢ $\frac{1}{2}\cos 2\theta$
- ▢ $\cos 2\theta$
- ▢ $2\cos\theta$

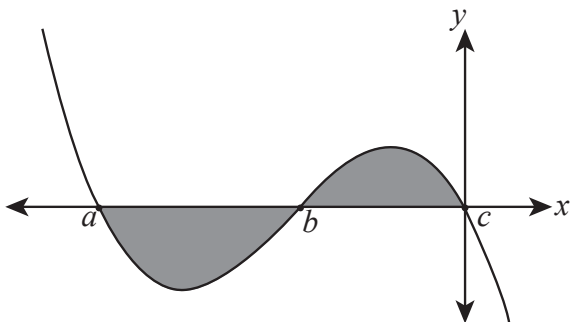
9. $\int \frac{dt}{\pi^{-5}} =$

- ▢ $\frac{\pi^6}{6} + c$
- ▢ $5\pi^4 + c$
- ▢ $\pi^5 t + c$
- ▢ $\frac{\pi^6}{6} t + c$

10. $\int \frac{x^2 - 9}{x + 3}\, dx =$

- ▢ $\frac{x^2}{2} + 3x + c$
- ▢ $\frac{x^2}{2} - 3x + c$
- ▢ $x^2 - 3x + c$
- ▢ $x^2 + 3x + c$

11. Consider the sketch, if $\int_a^b f(x)\,dx = -4$ and $\int_b^c f(x)\,dx = 3.7$, then the area bounded by the curve of $f(x)$ and the $x-$axis from $x = a$ to $x = c$ is:



- ▢ 0.3
- ▢ 0.7
- ▢ 3.7
- ▢ 7.7

12. If $\int_a^3 -2x\,dx = 16$, then $a =$

- ▢ ± 1
- ▢ $\pm\sqrt{17}$
- ▢ $\pm\sqrt{7}$
- ▢ ± 5

13. If A and B are independent events, $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $P(A' \cap B) =$

- ▢ $\frac{1}{12}$
- ▢ $\frac{1}{6}$
- ▢ $\frac{1}{4}$
- ▢ $\frac{11}{12}$

14. If E_1, E_2 and E_3 are three mutually exclusive events, $P(E_1) = 2P(E_2) = 0.32$ and $P(E'_1 \cap E'_2 \cap E'_3) = 0.21$, then $P(E_3) =$

- ▢ 0.15
- ▢ 0.31
- ▢ 0.52
- ▢ 0.69

Extended Questions

Write your answer for each of the three questions in the constructed response section in the space provided.

Be sure to show all your work, including the correct units where applicable.

Question Two: (14 marks)

A) i) If $\frac{6 - 9x}{(x + 4)(x^2 + 5)} = \frac{A(x^2 + 5) + Bx(x + 4) + c(x + 4)}{(x + 4)(x^2 + 5)}$, find A. (3 marks)

ii) Express $\frac{x^4 - 4x^2 + 1}{x(x-2)(x+2)}$ in partial fractions. (3 marks)

B) Find the equation of the normal to $y = x^2$ at the point where $x = 1$. (3 marks)

C) Without using calculator:
Find the value of cos 15° – tan 135°. (5 marks)

Question Three: [14 marks]

A) i) Find, from the first principles, the derivative of $3x$. (2 marks)

ii) Show that there is only one point on the curve $y = \frac{8}{3}x^3 + 4x^2 + 2x$, where the tangent is parallel to $y = 4$. (3 marks)

B) i) Find the range of values of x, for wich y is increasing, $y = -x^3 + 27x$. (3 mark)

ii) The sum of two natural numbers equals 120, in wich the multiplication of the square of one of them by the other number is to be **as maximum as possible**, Find the two numbers. (3 marks)

C) i) Find $\int \sqrt[5]{x^4}\, dx$. (2 marks)

ii) For all x if the second derivative of a curve equals 4, and it's gradient at (4,1) is 12, find the equation of the curve. (2 marks)

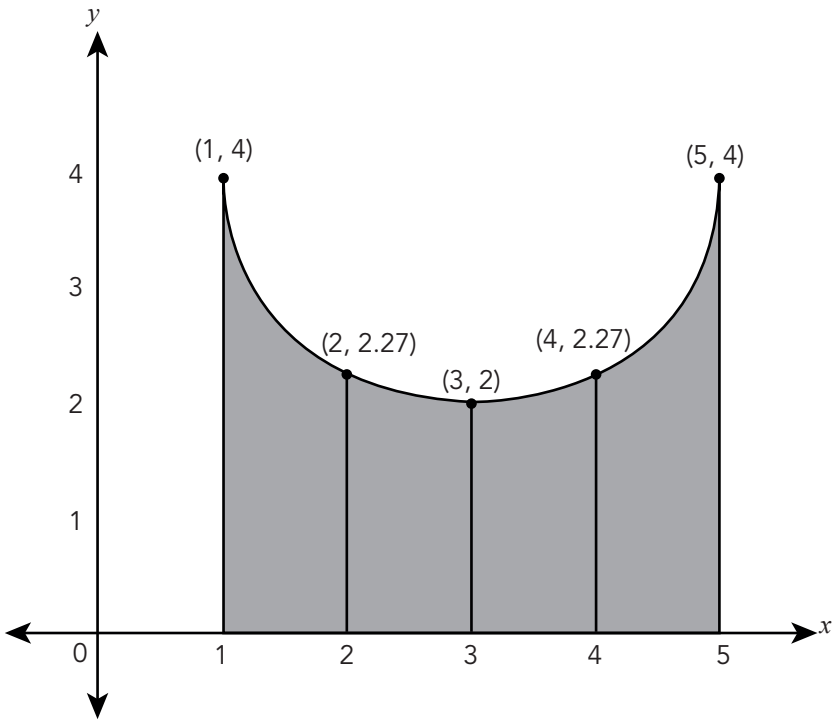
Question Four: (14 marks)

A) i) Find the value of R and $\tan\alpha$ in the identity:
$5\sin\theta + 3\cos\theta = R\cos(\theta - \alpha)$ (3 marks)

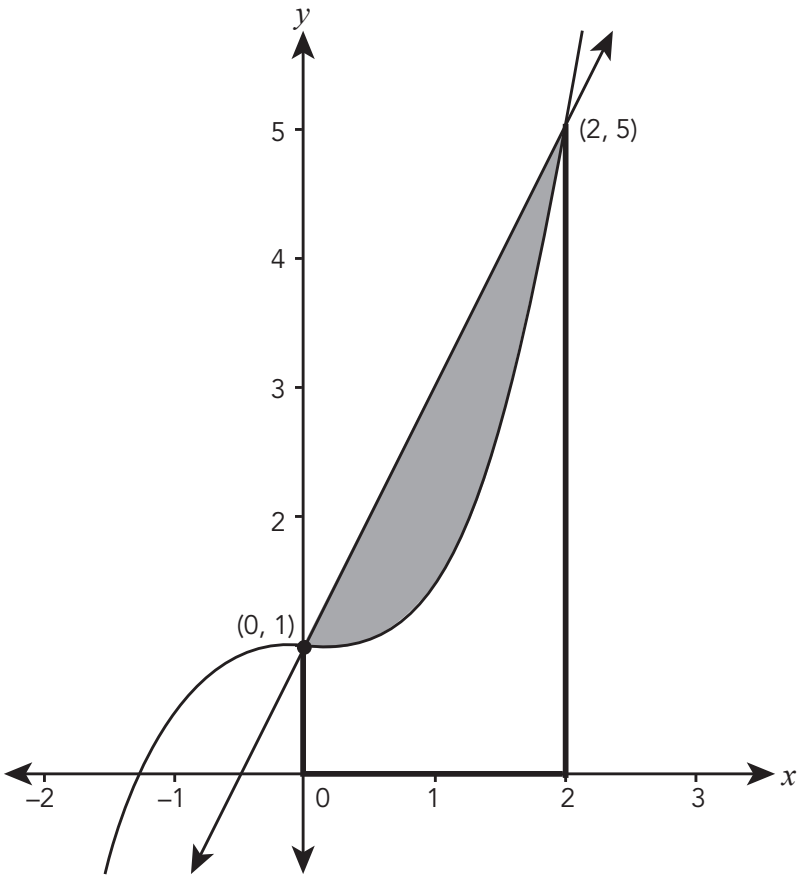
ii) Prove the following identity: (3 marks)

$$\left(\frac{4\sin^2(90° - \theta)}{\operatorname{cosec}^2\theta - \cot^2\theta} + \frac{8\cos^2(90° - \theta)}{2\sec^2\theta - 2\tan^2\theta}\right)\tan 2\theta = \frac{8\tan\theta}{2 - \sec^2\theta}$$

B) i) Consider the sketch. Find an approximation to the shaded area by using the Trapezium Rule. (2 marks)



ii) Consider the sketch. Find the shaded area. (2 marks)



C) A and B are defined in sample space such that, $P(A)$= 0.5, $P(B)$ = 0.4, $P(A \cup B)$ = 0.8. Find:

i) $P(A')$ (1 marks)

ii) $P(A \mid B)$

(3 marks)

END OF EXAMINATION AND GOOD LUCK!

Formulae Sheet For First Semester

Differentiation:

1. $y = x^n$ $\qquad \frac{dy}{dx} = nx^{(n-1)}$

2. Area and volume of a cuboid with length, width and height as *l*, *w*, and *h* respectively.
   $Area = 2lw + 2wh + 2lh$ $\qquad Volume = l \times w \times h$

3. Area and volume of a cylinder with radius, r, and height, *h.*
   $Area = 2\pi rh + 2\pi r^2$ $\qquad Volume = \pi r^2 h$

4. Area and volume of a sphere with radius, r.
   $Area = 4\pi r^2$ $\qquad Volume = \frac{4}{3}\pi r^3$

Trigonometry:

Pythagorean Formulas:

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A = 1 + \tan^2 A$
3. $\text{cosec}^2 A = 1 + \cot^2 A$
4. $\tan\theta = \cot(90 - \theta)$

Compound Angle Formulas:

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Double Angle Formulas:

1. $\sin 2A = 2 \sin A \cos A$
2. $\cos 2A = \cos^2 A - \sin^2 A$
   $\cos 2A = 2\cos^2 A - 1$
   $\cos 2A = 1 - 2\sin^2 A$
3. $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Half Angle Formulas:

1. $\sin^2 \frac{1}{2}A = \frac{1}{2}(1 - \cos A)$
2. $\cos^2 \frac{1}{2}A = \frac{1}{2}(1 + \cos A)$

The form $a\cos\theta + b\sin\theta$:

$a\cos\theta + b\sin\theta$ can be expressed in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ where $R = \sqrt{a^2 + b^2}$, $\alpha = \arctan\frac{b}{a}$.

Integration:

1. $\int x^n\,dx = \frac{x^{(n+1)}}{n+1} + c, \quad n \neq -1$

2. Area and volume of solids of revolution

$Area = \int_a^b f(x)dx$ $\qquad$ $Volume = \int_a^b \pi\,(f(x))^2 dx$

3. Trapezium rule

$Area = \frac{1}{2}h\left[y_0 + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right) + y_n\right]$

Probability:

1. Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Conditional Probability:

$$P(A\ given\ B) = P(A|B) = \frac{P(A \cap B)}{P(B)}$$

3. Multiplication Rule:

$$P(A \cap B) = P(A|B) \times P(B) \quad or \quad P(B|A) \times P(A)$$

4. Independent Rule:

A and B are independent if :

$$P(A|B) = P(A) \quad or \quad P(B|A) = P(B) \quad or \quad P(A \cap B) = P(A) \times P(B)$$

5. Mutually Exclusive Rule:

A and B are Mutually Exclusive if :

$$P(A \cap B) = 0$$

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Marking Guide Of First Semester Examination – Mathematics
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(Multiple choice)

Answer	Mark
Answer For Question One:	**14 × 2 = 28 marks**

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	a	a	d	c	b	a	d	c	c	b	d	d	c	b
Page	152	164	234	180	48	49 +52	50	70	167+ 169	169+ 173	328	328	96	95

(Extended Questions)

Answer	Mark	Page
QUESTION TWO (14 marks)		
A. i) (3marks)		
$\frac{6-9x}{(x+4)(x^2+5)} = \frac{A(x^2+5)+Bx(x+4)+c(x+4)}{(x+4)(x^2+5)}$		
$A(x^2+5)+Bx(x+4)+c(x+4) = 6-9x$	1	
$let\ x = -4$		
$A(16+5)+0+0$ =6-9(-4)	1	
$21A = 6+36$		
$21A = 42$		
$A = 2$	1	

(Extended Questions)		
Answer	**Mark**	**Page**
A. ii) (3 marks)		
$\dfrac{x^4-4x^2+1}{x(x-2)(x+2)}$		
$x^3-4x \overline{)\, x^4-4x^2+1}$ x x^4-4x^2 1	$\frac{1}{2}$	
$\dfrac{x^4-4x^2+1}{x(x-2)(x+2)} = x+\dfrac{1}{x(x-2)(x+2)}$ $= x+\dfrac{A}{x}+\dfrac{B}{(x-2)}+\dfrac{C}{(x+2)}$		
$\dfrac{1}{x(x-2)(x+2)} = \dfrac{A}{x}+\dfrac{B}{(x-2)}+\dfrac{C}{(x+2)} = \dfrac{A(x-2)(x+2)+Bx(x+2)+Cx(x-2)}{x(x-2)(x+2)}$	$\frac{1}{2}$	
$1 = A(x-2)(x+2)+Bx(x+2)+Cx(x-2)$ let $x=0$ $-4A=1 \Rightarrow A=-\dfrac{1}{4}$	$\frac{1}{2}$	
let $x=-2$ $8C=1 \Rightarrow C=\dfrac{1}{8}$	$\frac{1}{2}$	
let $x=2$ $8B=1 \Rightarrow B=\dfrac{1}{8}$	$\frac{1}{2}$	
So $\dfrac{x^4-4x^2+1}{x(x-2)(x+2)} = x-\dfrac{1}{4x}+\dfrac{1}{8(x-2)}+\dfrac{1}{8(x+2)}$	$\frac{1}{2}$	

(Extended Questions)		
Answer	**Mark**	**Page**
QUESTION TWO (14 marks)		
B. (3 marks)		
When $x = 1$, $y = 1^2 = 1$	$\frac{1}{2}$	
$y' = 2x$	$\frac{1}{2}$	
the gradient of tangent at $x = 1$, $y' = 2(1) = 2$	$\frac{1}{2}$	
the gradient of normal at $x = 1$ is $\frac{-1}{2}$	$\frac{1}{2}$	
the equation of the normal is $y - y_1 = m(x - x_1)$		
$y - 1 = \frac{-1}{2}(x - 1) \longrightarrow y = \frac{-1}{2}x + \frac{3}{2}$	1	
C. i) (5 marks)		
$cos15^\circ - tan135^\circ = \cos(60^\circ - 45^\circ) - \tan(180^\circ - 45^\circ)$	1	
$= \cos 60^\circ \cos 45^\circ + \sin 60^\circ \sin 45^\circ - \frac{tan\,180^\circ - tan\,45^\circ}{1 - \tan 180^\circ \times \tan 45^\circ}$	2	
$= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{0-1}{1-(0)(1)}\right)$	1	
$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}} + 1$	$\frac{1}{2}$	
$= \frac{1+\sqrt{3}}{2\sqrt{2}} + 1 = \frac{1+\sqrt{3}}{2\sqrt{2}} + \frac{2\sqrt{2}}{2\sqrt{2}} = \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$	$\frac{1}{2}$	
Another solution		
$cos15^\circ - tan135^\circ = \cos(45^\circ - 30^\circ) - \tan(90^\circ + 45^\circ)$	1	
$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ - \frac{\sin(90^\circ+45^\circ)}{cos(90^\circ+45^\circ)}$	2	
$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ - \frac{-\sin 45^\circ}{cos\,45^\circ}$	$\frac{1}{2}$	
$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - (-1)$	1	
$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} + 1 == \frac{1+\sqrt{2}+\sqrt{3}}{2\sqrt{2}}$	$\frac{1}{2}$	

(Extended Questions)		
Answer	**Mark**	**Page**
QUESTION THREE (14 marks)		
A. i) (2 marks) $\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $= \lim_{h\to 0} \frac{3(x+h)-3x}{h}$ $= \lim_{h\to 0} \frac{3x+3h-3x}{h}$ $= \lim_{h\to 0} \frac{3h}{h} = 3$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}$	
A. ii) (3 marks) $y' = \frac{8}{3}(3x^2) + 4(2x) + 2$ $y' = \frac{8}{3}(3x^2) + 4(2x) + 2 = 0$ $8x^2 + 8x + 2 = 0$ $2(4x^2 + 4x + 1) = 0$ the gradient is zero $\longrightarrow (2x+1)^2 = 0$ $x = \frac{-1}{2}$	1 $\frac{1}{2}$ $\frac{1}{2}$ 1	
B. i) (3 marks) $y' = -3x^2 + 27$ $y' > 0$ $27 - 3x^2 > 0$ y is increasing function if $3(9 - x^2) > 0$ $(3-x)(3+x) > 0$ $x = 3 \quad , \quad x = -3$ y is increasing function of x for $-3 < x < 3$	1 $\frac{1}{2}$ $\frac{1}{2}$ 1	

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(Extended Questions)		
Answer	**Mark**	**Page**
B. ii) (3 marks) Let the first number be x Let the second number be y $x + y = 120$ $y = (120 - x)$ $f(x) = x^2\, y$ $f(x) = x^2(120 - x)$ $f(x) = 120\, x^2 - x^3$ $f'(x) = 240x - 3x^2$ $240\, x - 3x^2 = 0 \longrightarrow 3x(80 - x) = 0$ $x = 0$ (rejected) , $x = 80$ $f''(x) = 240 - 6x$ $f''(80) = 240 - 6(80) = -240 < 0$ $then\ y = 120 - 80 = 40$ The numbers are: 80,40	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	
C. i) (2 marks) $= \int x^{\frac{4}{5}}\, dx$ $\frac{x^{\frac{4}{5}+1}}{\frac{4}{5}+1} + c = \frac{x^{\frac{9}{5}}}{\frac{9}{5}} + c$ $\frac{5\, x^{\frac{9}{5}}}{9} + c$	$1 + \frac{1}{2}$ $\frac{1}{2}$	

(Extended Questions)		
Answer	**Mark**	**Page**
C. ii) (2 marks) $f''(x) = 4$ $f'(x) = \int 4\,dx = 4x + c$ $f'(4) = 4(4) + c = 12$ $c = -4$ $f'(x) = 4x - 4$ $f(x) = \int (4x - 4)dx$ $f(x) = 2x^2 - 4x + c$ $f(4) = 2(4)^2 - 4(4) + c = 1$ $c = -15$ $f(x) = 2x^2 - 4x - 15$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	
QUESTION FOUR (14 marks) **A. ii) (3 marks)** $5\sin\theta + 3\cos\theta = R\cos(\theta - \alpha)$ $a = 3, b = 5$ $R = \sqrt{a^2 + b^2} = \sqrt{9 + 25} = \sqrt{34}$ $\tan\alpha = \frac{b}{a} = \frac{5}{3}$	$\frac{1}{2} + \frac{1}{2}$ $\frac{1}{2} + \frac{1}{2}$ 1	
A. ii) (3 marks) $\left[\frac{4\cos^2\theta}{1} + \frac{8\sin^2\theta}{2(1)}\right] \times \frac{2\tan\theta}{1-\tan^2\theta}$ $= 4\,(\sin^2\theta + \cos^2\theta) \times \frac{2\tan\theta}{1-(-1+\sec^2\theta)}$ $= 4\,(1) \times \frac{2\tan\theta}{1+1-\sec^2\theta}$ $= \frac{8\tan\theta}{2-\sec^2\theta}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$	

(Extended Questions)		
Answer	**Mark**	**Page**
B. i) (2 marks) **Area** $\cong \frac{h}{2}\,[\,y_0 + y_n + 2(y_1 + y_2 + \cdots + y_{n-1})$ $\cong \frac{1}{2}\,[\,4 + 4 + 2(2.27 + 2 + 2.27)]$ $\cong \frac{1}{2}\,(21.08) = 10.54$ square unit	1 1	
B. ii) (2 marks) The area of trapezium $= \frac{1}{2}(5+1) \times 2 = 6$ square unit The area under the curve f(x) is: $\int_0^2 (\frac{1}{2}\,x^3 + 1)\,dx = [\frac{1}{2}(\frac{x^4}{4}) + x\,]_0^2$ $= [\frac{x^4}{8} + x\,]_0^2 = (\frac{16}{8} + 2) - 0 = 4$ square unit The shaded area = The area of trapezium - The area under the curve f(x) $= 6 - 4 = 2$ square unit	$\frac{1}{2}$ 1 $\frac{1}{2}$	
C. i) (1 marks) P(A') = 1 - P(A) $= 1 - 0.5$	1	
C. ii) (3 marks) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $P(A \cap B) = 0.5 + 0.4 - 0.8 = 0.1$ $P(A|B) = \frac{P(A \cap B)}{P(B)}$ $= \frac{0.1}{0.4}$ $= \frac{1}{4} = 0.25$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$	

((End of the Marking Guide))