رقم الورقة	
رقم المغلف	

حاضر ☐

غائب ☐

سلطنة عمان
وزارة التربية والتعليم

امتحان شهادة دبلوم التعليم العام للمدارس الخاصة (ثنائية اللغة)

للعام الدراسي ١٤٣٣/١٤٣٤ هـ - ٢٠١٢ / ٢٠١٣ م

الدور الأول - الفصل الدراسي الأول

تنبيه:
- الرياضيات.
- الأسئلة في (١٦) صفحات.
- زمن الإجابة: ثلاث ساعات.
- الإجابة في الورقة نفسها.

تعليمات وضوابط التقدم للامتحـان:

- الحضور إلى اللجنة قبل عشر دقائق من بدء الامتحان للأهمية.
- إبراز البطاقة الشخصية لمراقب اللجنـة.
- يمنع كتابة رقم الجلوس أو الاسم أو أي بيانات أخرى تدل على شخصية الممتحن في دفتر الامتحان، وإلا ألغي امتحانه.
- يحظر على الممتحنين أن يصطحبوا معهم بمركز الامتحان كتبا دراسية أو كراسات أو مذكرات أو هواتف محمولة أو أجهزة النداء الآلي أو أي شيء له علاقة بالامتحان كما لا يجوز إدخال آلات حادة أو أسلحة من أي نوع كانت أو حقائب يدوية أو آلات حاسبة ذات صفة تخزينية.
- يجب أن يتقيد المتقدمون بالزي الرسمي (الدشداشة البيضاء والمصر أو الكمة للطلاب والدارسين والزي المدرسي للطالبات واللباس العماني للدارسات) ويمنع النقاب داخل المركز ولجان الامتحان.
- لا يسمح للمتقدم المتأخر عن موعد بداية الامتحان بالدخول إلا إذا كان التأخير بعذر قاهر يقبله رئيس المركز وفي حدود عشر دقائق فقط.
- يتم الالتزام بالإجراءات الواردة في دليل الطالب لأداء امتحان شهادة دبلوم التعليم العام.
- يقوم المتقدم بالإجابة عن أسئلة الامتحان المقالية بقلم الحبر (الأزرق أو الأسود).
- يقوم المتقدم بالإجابة عن أسئلة الاختيار من متعدد بتظليل الشكل (☐) وفق النموذج الآتي:

س - عاصمـة سلطنة عمـــان هي:

☐ القاهرة ☐ الدوحة

■ مسقط ☐ أبوظبي

ملاحظة: يتم تظليل الشكل (■) باستخدام القلم الرصاص وعند الخطأ، امسح بعناية لإجراء التغيير.

صحيح ■ **غير صحيح** ☐ ◉ ◒ ✗ ✓

Question One (28 marks)

There are 14 multiple–choice items worth two marks each.
Shade the correct answer for each of the following items.

1. $\lim_{h \to 0} \dfrac{f(-2) - f(-2+h)}{h} =$

- ☐ $f'(2)$
- ☐ $-f'(2)$
- ☐ $f'(-2)$
- ☐ $-f'(-2)$

2. If $y = a\,x^2 + 5$ and $\dfrac{d^2y}{dx^2} = 6$ at $x = -1$, then $a =$

- ☐ -3
- ☐ -1
- ☐ 1
- ☐ 3

3. The coordinates of the stationary point of the curve $y = 2x - x^2$ is:

- ☐ $(1, -2)$
- ☐ $(0, 0)$
- ☐ $(1, 1)$
- ☐ $(2, 0)$

4. If $\dfrac{6x}{(x-1)(x+2)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x+2)}$, then the value of $2A - B$ is:

- ☐ 0
- ☐ 2
- ☐ 4
- ☐ 6

5. Which of trigonometric functions are both odd?

- ☐ $cos\theta$, $cosec\theta$
- ☐ $cosec\theta$, $cot\theta$
- ☐ $sec\theta$, $cot\theta$
- ☐ $cos\theta$, $sec\theta$

6. If $cot\theta = \frac{4}{3}$ and θ is reflex, then $sec\theta =$

- ☐ $\frac{-5}{3}$
- ☐ $\frac{-5}{4}$
- ☐ $\frac{5}{4}$
- ☐ $\frac{5}{3}$

7. If $cot(3\theta - 30°) = \frac{1}{\sqrt{3}}$, $0° < \theta < 90°$, then $\theta =$

- ☐ $10°$
- ☐ $20°$
- ☐ $30°$
- ☐ $60°$

8. If $t = cos\theta$, then $t^2 - \frac{1}{2} =$

- ☐ $cos2\theta$
- ☐ $\frac{1}{2} cos2\theta$
- ☐ $cos\frac{\theta}{2}$
- ☐ $\frac{1}{2} cos\frac{\theta}{2}$

9. $\int (3\pi^2 - 3)\, dt =$

- ☐ $\pi^3 t - 3\pi t + c$
- ☐ $3\pi^2 t - 3t + c$
- ☐ $3\pi^3 - 3\pi + c$
- ☐ $\pi^3 - 3\pi + c$

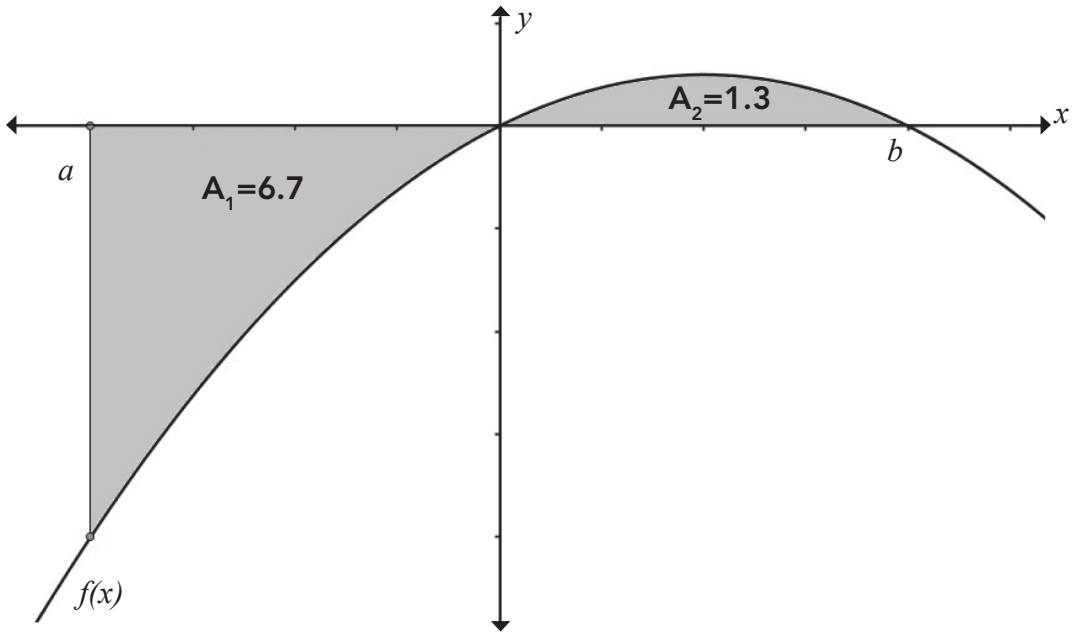
10. $\int \frac{x^3 - 8}{x - 2}\, dx =$

- ☐ $\frac{x^3}{3} + x^2 + 4x + c$
- ☐ $\frac{x^3}{3} - x^2 + 4x + c$
- ☐ $\frac{x^3}{3} + 2x^2 + 4x + c$
- ☐ $x^3 - 2x^2 + 4x + c$

11. Consider the sketch,

If A_1 ,A_2 are two areas, then $\int_a^b f(x)dx =$

- ☐ 8
- ☐ 5.4
- ☐ – 5.4
- ☐ – 8



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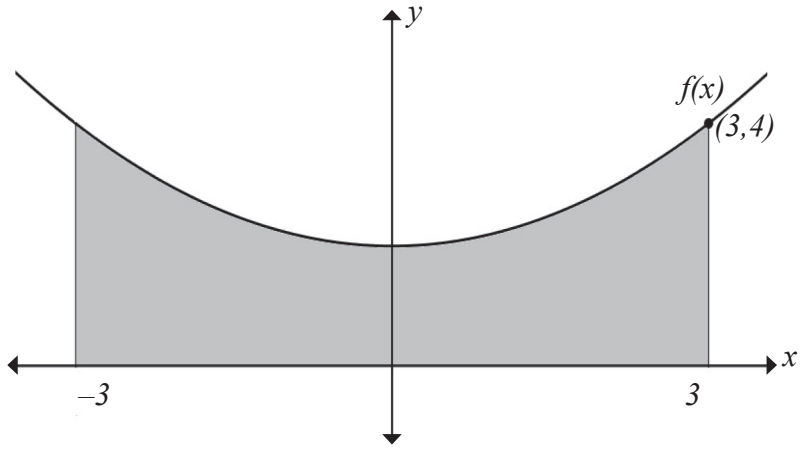
12. Consider the sketch. It's symmetric around *y-axis*. If the sum of ordinates's values ($y_1, y_2, \dots, y_{n-1}$) is 5 and $\int_0^3 f(x)dx=3$, then the width of each interval for the shaded area is:

- ☐ $\frac{1}{3}$
- ☐ $\frac{3}{5}$
- ☐ $\frac{2}{3}$
- ☐ $\frac{6}{5}$



13. If E_1 and E_2 are two mutually exclusive events, $P(E_1) = 0.05$, $P(E_2')= 0.07$, then $P(E_1 \cup E_2)$ =

- ☐ *0.02*
- ☐ *0.12*
- ☐ *0.35*
- ☐ *0.98*

14. On an experiment of throwing a fair die (has each face number 1 to 6) and tossing a coin, the results were recorded on each of them. If A is **"the event of observing tail"**, B is **"the event of observing 3"** , then P(A ∪ B) =

- ☐ $\frac{1}{12}$
- ☐ $\frac{1}{3}$
- ☐ $\frac{7}{12}$
- ☐ $\frac{2}{3}$

Extended Questions

Write your answer for each of the three questions in the constructed response section in the space provided. Be sure to show all your work and correct units where applicable.

Question Two: [14 marks]

a) i. If $\frac{5x+7}{(x-5)(x^2+7)} = \frac{A(x^2+7) + Bx(x-5) + c\,(x-5)}{(x-5)(x^2+7)}$, find A. **(3 marks)**

ii. Express $\frac{x^3 + 4x^2}{(x + 1)(x + 3)}$ in partial fractions **(3marks)**

b) Find the equation of the tangent to $y = x^2 + x$ at $x = 1$ **(3marks)**

c) Without using a calculator:
Find the value of sin120° + tan75° **(5marks)**

Question Three: [14 marks]

a) **i.** If $f(x) = x^{\frac{1}{4}}$, find $f''(x)$ **(2 marks)**

ii. Given that $y = 2x^3 + x$ has gradient equal 7 at the point $(a\ ,\ b)$, find possible values for a and b. **(2 marks)**

b) i. Find the range of values of x for which y is decreasing, given that $y = \frac{4}{3}x^3 - 16x + 9$.

(3 marks)

ii. A container in the shape of a right circular cylinder with no top. It has surface area 3π square metres. What height (h) and base radius (r) which makes the volume of the container as maximum as possible? **(3 marks)**.

c) **i.** Find $\int (\sqrt[3]{y^5} - 8)dy$ **(2 marks)**

ii. Find the equation of the curve which its gradient is given by $3x^2 - 2x$ and f(2) = 7 **(2 marks)**

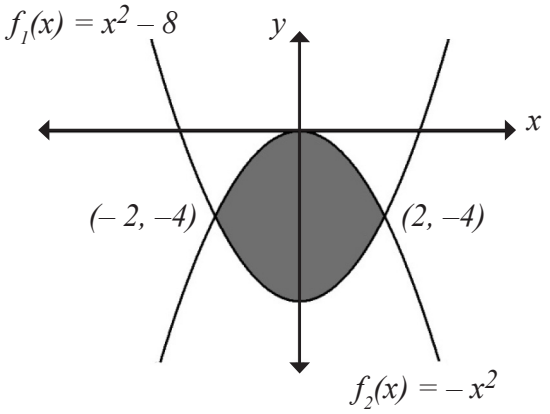
Question Four: [14 marks]

a) **i.** Find the value of R and tan α in this identity:
$4\sin\theta + 2\cos\theta = R\cos(\theta - \alpha)$ **(3 marks)**

ii. Prove the identity $\sqrt{\dfrac{2\cot^2(90° - \theta) - 6 + 6\sec^2\theta}{-2 + 2\operatorname{cosec}^2\theta}} = \pm 2\tan^2\theta.$ **(3 marks)**

b) i. If f(6) = 13 and f(9) = 17, find $\int_6^9 f'(x)\,dx$ **(2 marks)**

ii. Consider the sketch. Find the shaded area. **(2 marks)**



c) If A and B are defined in the sample space, $P(A \cup B) = \frac{3}{4}$, $P(A) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{4}$, find:

i. $P(A')$. **(1 mark)**

ii. $P(A \mid B)$. **(3 marks)**

[End of Examination]

Formulae Sheet For First Semester

Differentiation:

1. $y = x^n$ $\qquad \frac{dy}{dx} = nx^{(n-1)}$

2. Area and volume of a cuboid with length, width and height as l, w, and h respectively.
   $Area = 2lw + 2wh + 2lh$ $\qquad Volume = l \times w \times h$

3. Area and volume of a cylinder with radius, r, and height, h.
   $Area = 2\pi rh + 2\pi r^2$ $\qquad Volume = \pi r^2 h$

4. Area and volume of a sphere with radius, r.
   $Area = 4\pi r^2$ $\qquad Volume = \frac{4}{3}\pi r^3$

Trigonometry:

Pythagorean Formulas:

1. $\sin^2 A + \cos^2 A = 1$
2. $\sec^2 A = 1 + \tan^2 A$
3. $\text{cosec}^2 A = 1 + \cot^2 A$
4. $\tan\theta = \cot(90 - \theta)$

Compound Angle Formulas:

1. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Double Angle Formulas:

1. $\sin 2A = 2 \sin A \cos A$
2. $\cos 2A = \cos^2 A - \sin^2 A$
   $\cos 2A = 2\cos^2 A - 1$
   $\cos 2A = 1 - 2\sin^2 A$
3. $\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

Half Angle Formulas:

1. $\sin^2 \frac{1}{2}A = \frac{1}{2}(1 - \cos A)$
2. $\cos^2 \frac{1}{2}A = \frac{1}{2}(1 + \cos A)$

The form $a\cos\theta + b\sin\theta$:

$a\cos\theta + b\sin\theta$ can be expressed in the form $R\cos(\theta \pm \alpha)$ or $R\sin(\theta \pm \alpha)$ where $R = \sqrt{a^2 + b^2}$, $\alpha = \arctan\frac{b}{a}$.

Integration:

1. $\int x^n \, dx = \frac{x^{(n+1)}}{n+1} + c, \quad n \neq -1$

2. Area and volume of solids of revolution

$$Area = \int_a^b f(x)dx$$

3. Trapezium rule

$$Area = \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + \cdots + y_{n-1}\right)\right]$$

Probability:

1. Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

2. Conditional Probability:

$$P(A \text{ given } B) = P(A|B) = \frac{P(A \cap B)}{P(B)}$$

3. Multiplication Rule:

$$P(A \cap B) = P(A|B) \times P(B) \quad or \quad P(B|A) \times P(A)$$

4. Independent Rule:

A and B are independent if :

$$P(A|B) = P(A) \quad or \quad P(B|A) = P(B) \quad or \quad P(A \cap B) = P(A) \times P(B)$$

5. Mutually Exclusive Rule:

A and B are Mutually Exclusive if : $P(A \cap B) = 0$

مُسَوَّدَة، لا يتم تصحيحها

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(Multiple choice)

Answer	Mark
Answer For Question One:	**14 × 2 = 28 marks**

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14
	d	d	c	a	b	b	c	b	b	a	c	c	d	c
Page	152	157	234	180	48	49+ 52	50	68	167+ 169	170	329	342	95	97

(Extended Questions)

Answer	Mark	Page
QUESTION TWO (14 marks)		
A. i) (3marks)		
$\frac{5x+7}{(x-5)(x^2+7)} = \frac{A(x^2+7)+Bx(x-5)+c(x-5)}{(x-5)(x^2+7)}$		
$A(x^2+7)+Bx(x-5)+c(x-5)=5x+7$	1	
$Let\ x=5$		
$(25+7)A+0+0=(25+7)$	1	
$32A=32$		
$A=1$	1	
A. ii) (3 marks)		186
$\frac{x^3+4x^2}{(x+1)(x+3)}$		
$x^2+4x+3 \overline{)\, x^3+4x^2}$ $\quad x$		
x^3+4x^2+3x	$\frac{1}{2}$	
$-3x$		

(Extended Questions)		
Answer	**Mark**	**Page**
$\frac{x^3+4x^2}{(x+1)(x+3)} = x + \frac{-3x}{(x+1)(x+3)}$	$\frac{1}{2}$	186
$= x + \frac{A}{(x+1)} + \frac{B}{(x+3)}$		
$-3x = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$		
$-3x = A(x+3)+B(x+1)$	$\frac{1}{2}$	
let $x=-3$		
$9 = -2B \Rightarrow B = \frac{-9}{2}$	$\frac{1}{2}$	
let $x=-1$		
$3 = 2A \Rightarrow A = \frac{3}{2}$	$\frac{1}{2}$	
So $\frac{x^3+4x^2}{(x+1)(x+3)} = x + \frac{\frac{3}{2}}{(x+1)} - \frac{\frac{9}{2}}{(x+3)} = x + \frac{3}{2(x+1)} - \frac{9}{2(x+3)}$	$\frac{1}{2}$	
Other solution to find A and B:		
By coefficient		
$A + B = -3$		
$-\ 3A + B = 0$		
$2A = 3$	$\frac{1}{2}$	
$A = \frac{3}{2}$		
$B = -3 - A$		
$= -3 - \frac{3}{2} = \frac{-9}{2}$	$\frac{1}{2}$	
B. (3 marks)		
at $x = 1$: $y = 1^2 + 1 = 2 \rightarrow (1, 2)$	$\frac{1}{2}$	161
$f'(x) = 2x + 1$	1	
Gradient at $x = 1$		
$f'(x) = 2(1) + 1 = 3$	$\frac{1}{2}$	
The equation of tangent to y is $y - y_1 = m(x - x_1)$		
$y - 2 = 3(x - 1)$	$\frac{1}{2}$	
$y = 3x - 1$	$\frac{1}{2}$	

(Extended Questions)		
Answer	**Mark**	**Page**
C. i) (2 marks)		60
$\sin 120^\circ + \tan 75^\circ = sin\, 2(60^\circ) + \tan(30^\circ + 45^\circ)$	1	
$= 2\sin 60^\circ \cos 60^\circ + \frac{tan\, 30^\circ + tan\, 45^\circ}{1 - \tan 30^\circ \times \tan 45^\circ}$	2	
$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) + \frac{\frac{1}{\sqrt{3}}+1}{1-\left(\frac{1}{\sqrt{3}}\right)(1)}$	1	
$= \frac{\sqrt{3}}{2} + \frac{\frac{1}{\sqrt{3}}+1}{1-\left(\frac{1}{\sqrt{3}}\right)}$	$\frac{1}{2}$	
$= \frac{\sqrt{3}}{2} + \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}}{2} + \frac{1+\sqrt{3}}{\sqrt{3}-1}$	$\frac{1}{2}$	
$= \frac{\sqrt{3}}{2} + \frac{(\sqrt{3}+1)^2}{2} = \frac{3\sqrt{3}+4}{2}$		
Another solution		
$\sin 120^\circ + \tan 75^\circ = sin\,(90^\circ + 30^\circ) + \tan(30^\circ + 45^\circ)$	1	
$= (\sin 90^\circ \cos 30^\circ + \cos 90^\circ \sin 30^\circ) + \frac{tan\, 30^\circ + tan\, 45^\circ}{1 - \tan 30^\circ \times \tan 45^\circ}$	2	
$= (1\left(\frac{\sqrt{3}}{2}\right) + 0\left(\frac{1}{2}\right)) + \frac{\frac{1}{\sqrt{3}}+1}{1-\left(\frac{1}{\sqrt{3}}\right)(1)}$	1	
$= \frac{\sqrt{3}}{2} + \frac{\frac{1+\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}}{2} + \frac{1+\sqrt{3}}{\sqrt{3}-1}$	$\frac{1}{2}$	
$= \frac{\sqrt{3}}{2} + \frac{(\sqrt{3}+1)^2}{2} = \frac{3\sqrt{3}+4}{2}$	$\frac{1}{2}$	
QUESTION THREE (14 marks)		157
A. i) (2 marks)		
$f'(x) = \frac{1}{4}x^{\frac{-3}{4}}$	1	
$f''(x) = \left(\frac{1}{4}\right)\left(\frac{-3}{4}\right)x^{\frac{-7}{4}} = \frac{-3}{16}x^{\frac{-7}{4}}$	1	

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(Extended Questions)		
Answer	**Mark**	**Page**
A. ii) (2 marks)		
$y' = 6x^2 + 1$		159
$at(a,b)$: $y' = 6a^2 + 1 = 7$ ..... (1)	$\frac{1}{2}$	+163
$6a^2 = 6$	$\frac{1}{2}$	
$a = \pm 1$		
$at(a,b)$: $b = 2b^3 + a$		
if $a = 1$ *then* $b = 2(1)^3 + 1 = 3$	$\frac{1}{2}$	
if $a = -1$ *then* $b = 2(-1)^3 + (-1) = -3$	$\frac{1}{2}$	
B. i) (3 marks)		**231**
$y' = 3(\frac{4}{3})x^2 - 16 \longrightarrow y' = 4x^2 - 16$	1	
y is decreasing function if $y' < 0$		
$4x^2$-16 < 0	$\frac{1}{2}$	
$4(x^2$- 4) < 0		
$4(x-2)(x+2) < 0$	$\frac{1}{2}$	
y is decreasing function of x for -2 $< x <$ 2	1	
ii) (3 marks)		
A= πr^2+2 π rh		
3 π= πr^2+2 π rh		
3 = r^2+2 rh		
2 rh= 3 − r^2		
h =$\frac{3-r^2}{2r}$ ..... (1)	$\frac{1}{2}$	
V= π r^2h		
V= π ($\frac{3-r^2}{2r}$)r^2	$\frac{1}{2}$	
V' =$\frac{\pi}{2}$(3 r − r^3)	$\frac{1}{2}$	
$\frac{\pi}{2}$(3 − 3r^2)= 0	$\frac{1}{2}$	
3(1 − r^2)=0		
r=1 , r=-1 (rejected)	$\frac{1}{2}$	
V"=-3rπ .....maximum volume where r =1		
h=$\frac{3-1}{2}$=1	$\frac{1}{2}$	

(Extended Questions)

Answer	Mark	Page
C. i) (2 marks)		170
$= \int (y^{\frac{5}{3}} - 8)\, dy$		
$= \frac{y^{\frac{5}{3}+1}}{\frac{5}{3}+1} - 8y + c$	1	
$= \frac{3y^{\frac{8}{3}}}{8} - 8y + c$	1	
ii) (2 marks)		171 + 172
$f(x) = \int (3x^2 - 2x)\, dx$	$\frac{1}{2}$	
$= x^3 - x^2 + c$	$\frac{1}{2}$	
$7 = 2^3 - 2^2 + c$		
$7 = 4 + c$		
$c = 3$	$\frac{1}{2}$	
$y = x^3 - x^2 + 3$	$\frac{1}{2}$	
QUESTION FOUR (14 marks)		
A. i) (3 marks)		76 + 78
$4\sin\theta + 2\cos\theta = R\cos(\theta - \alpha)$		
$a = 2, b = 4$	$\frac{1}{2} + \frac{1}{2}$	
$R = \sqrt{a^2 + b^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$	$\frac{1}{2} + \frac{1}{2}$	
$\tan\alpha = \frac{b}{a} = \frac{4}{2} = 2$	$\frac{1}{2} + \frac{1}{2}$	
ii) (3 marks)		
$= \sqrt{\frac{2\tan^2\theta + 6\tan^2\theta}{2\cot^2\theta}}$	$\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$	49
$= \sqrt{\frac{8\tan^2\theta}{2\cot^2\theta}}$	$\frac{1}{2}$	
$= 2\sqrt{\tan^4\theta}$	$\frac{1}{2}$	
$= \pm 2\tan^2\theta$	$\frac{1}{2}$	

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(Extended Questions)

Answer	Mark	Page
B. i) (2 marks)		
$\int_6^9 f'(x)\,dx = [f(x)]_6^9$	1	
$= f(9) - f(6)$	$\frac{1}{2}$	326
$= 17 - 13 = 4$	$\frac{1}{2}$	
ii) (2 marks)		
$A = \int_{-2}^{2}(-x^2 - x^2 + 8)dx = \int_{-2}^{2}(-2x^2 + 8)dx$	$\frac{1}{2}+\frac{1}{2}$	336
$= [\frac{-2}{3}x^3 + 8x]_{-2}^{2}$	$\frac{1}{2}$	
$= (\frac{-2}{3}(2)^3 + 16) - (\frac{-2}{3}(-2)^3 - 16)$		
$= \frac{32}{3} - \frac{-32}{3}$		
$= \frac{64}{3}$	$\frac{1}{2}$	
C. i) (1 mark)		
$P(A') = 1 - P(A)$	$\frac{1}{2}$	82
$= 1 - \frac{2}{3}$		
$= \frac{1}{3}$	$\frac{1}{2}$	
ii) (3 marks)		
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
$\frac{3}{4} = \frac{2}{3} + P(B) - \frac{1}{4}$	$\frac{1}{2}$	87
$P(B) = \frac{3}{4} - \frac{2}{3} + \frac{1}{4}$	$\frac{1}{2}$	
$= \frac{4}{12} = \frac{1}{3}$	$\frac{1}{2}$	
$P(A\|B) = \frac{P(A \cap B)}{P(B)}$		
$= \frac{\frac{1}{4}}{\frac{1}{3}}$	1	
$= \frac{3}{4}$	$\frac{1}{2}$	

((End of the Marking Guide))